UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
International General Certificate of Secondary Education

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

0661845285

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/06**

Paper 6 (Extended) **May/June 2010**

1 hour 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Graphics Calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, highlighters, glue or correction fluid.
You may use a pencil for any diagrams or graphs.

Answer both parts **A** and **B**.
You are advised to spend 45 minutes on Part **A** and 45 minutes on Part **B**.

You must show all relevant working to gain full marks for correct methods, including sketches.

In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.

At the end of the examination, fasten all your work securely together.
The total number of the marks for this paper is 40.

This document consists of **7** printed pages and **1** blank page.

UNIVERSITY of CAMBRIDGE International Examinations

For Examiner's

Answer **both** parts **A** and **B**.

A. INVESTIGATION FERMAT'S LITTLE THEOREM (20 marks)

You are advised to spend 45 minutes on part **A**.

The division 46 ÷ 5 gives 9 with a remainder of 1.
A method for finding the remainder is

46 ÷ 5 = **9**.2
Because **9** × 5 = 45, the remainder is 46 – 45 = 1.

The division 921 ÷ 7 gives a remainder of 4.
A method for finding the remainder is

921 ÷ 7 = **131**.571….
Because **131** × 7 = 917, the remainder is 921 – 917 = 4.

The division $2^{11} \div 13$ gives a remainder of 7.
A method for finding the remainder is

$2^{11} \div 13 =$ **157**.5384….
Because **157** × 13 = 2041, the remainder is $2^{11} - 2041 = 7$.

1 Find the remainder in these divisions.

(a) 1234 ÷ 7

....

(b) $2^9 \div 9$

....

2 In 1640 the French mathematician Fermat found something interesting about the remainde dividing by a **prime** number. Some of his results are shown in the table below.

Prime	Division	Remainder	Division	Remainder	Division	Remainder
3	$2^3 \div 3$					
5	$2^5 \div 5$		$3^5 \div 5$		$4^5 \div 5$	
7	$2^7 \div 7$	2	$3^7 \div 7$		$4^7 \div 7$	
11	$2^{11} \div 11$					

Complete the unshaded boxes in this table. You may use the space below to show any working.

3 Use the patterns you have found in your table to complete the following statements.

(a) $7^{11} \div$ .... has a remainder of ....

(b) $8^{17} \div$ .... has a remainder of ....

4 From the table $2^7 \div 7$ has a remainder of 2.
So $2^7 - 2$ has a **prime** factor of 7.

Because $2^7 - 2 = 2(2^6 - 1)$ and 7 is not a factor of 2,
$2^6 - 1$ has a **prime** factor of 7.

(a) Complete the following statements to show why $5^{12} - 1$ has a prime factor of 13.

.... has a remainder of 5.

So $5^{13} - 5$ has a prime factor of ....

Because $5^{13} - 5 =$ .... and .... is not a factor of 5,

$5^{12} - 1$ has a **prime** factor of 13.

(b) **Write down** a prime factor of $8^{16} - 1$. ....

Part **A** continues on **page 4**.

5 Complete the general statement below.

$a^{p-1} - 1$ has a prime factor of ....

This is known as Fermat's Little Theorem.

6 Fermat noted that p must not be a factor of a.
Give an example to show the result is **not** true when p is a factor of a.

....

7 By writing $7^{24} - 1$ in different ways, you can use Fermat's Little Theorem to find prime factors.

Examples:

$$7^{24} - 1 = \left(7^{24}\right)^{1} - 1 = \left(7^{24}\right)^{2-1} - 1$$

Using Fermat's Little Theorem with $p = 2$, $7^{24} - 1$ has a prime factor of 2.

$$7^{24} - 1 = \left(7^{8}\right)^{3} - 1 = \left(7^{8}\right)^{4-1} - 1$$

Fermat's Little Theorem with $p = 4$ cannot be used because 4 is not a prime number.

Use the above method to find as many **prime** factors as you can of $7^{24} - 1$.
Remember: p must not be a factor of a.
Show all your working.

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Part **B** starts on **page 6**.

B. MODELLING CHANGE OF AVERAGE SPEED (20 marks)

You are advised to spend 45 minutes on part **B**.

Sam makes a journey in two stages.
In **Stage 1**, he cycles at 10 km/h for one hour.
In **Stage 2**, he cycles at 20 km/h.

1 In Stage 2, Sam cycles for 30 minutes.

(a) Find the **total** distance he travels.

.... km

(b) Show that his average speed for the whole journey is 13.3 km/h.

2 If, in Stage 2, he cycles for 15 minutes, show that his average speed is now 12 km/h.

3 If, in Stage 2, he cycles for 12 minutes, find his average speed.

.... km/h

4 **(a)** In Stage 2, Sam cycles for x minutes.
Write down a formula for S, the average speed, in km/h.

(b) Your formula is a mathematical model for the average speed.
Show that it simplifies to

$$S = \frac{600 + 20x}{60 + x}.$$

For Examiner's

5 Use the model to find Sam's average speed if, in Stage 2, he cycles for 13 minutes.

.... km/h

6 On the axes below, sketch the graph of S against x for $0 \leqslant x \leqslant 500$. Show any important details on your sketch.

7 Find how many minutes Sam must cycle in Stage 2 to have an average speed of 13 km/h.

.... min

Part **B** continues on **page 8**.

For Examiner's Use

8 In Stage 2, instead of 20 km/h, Sam cycles at y km/h for x minutes.

(a) Modify the model in **question 4(b)**.

(b) Find y if, after 24 minutes in Stage 2, Sam's average speed is 8 km/h.

$y =$ ....

(c) On the axes below sketch the graph of S against x if he **stops** for x minutes at the end of Stage 1.